Nonlinear Control of Twin Helicopter with Slung Load

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ABSTRACT

This paper focuses on simulation and control of a formation of two helicopters carrying a single external load. A simulation model of a rotorcraft twin-lift system is developed and implemented in the MATLAB / Simulink environment. All objects in the twin-helicopter system are coupled exclusively through elastic cable forces that act on each of the bodies. Analytical checks of the model are performed for the trim and stability characteristics of each helicopter, and results show reasonable and expected behavior due to the physical coupling from the cables. A non-linear control scheme is developed that uses an aerodynamic inverse approach which can expediently introduce cable forces and moments feedback in inner control loop. The controller calculates desired control inputs based on an inversion of the quasi-steady aerodynamics and the desired trajectory of each helicopter. In order to accurately track the commanded trajectory, the control law makes direct use of the measured cable force acting on each helicopter. The controller’s performance and robustness are demonstrated in non-linear simulation of the twin-lift system.

NOTATION

- \( X, Y, Z \) are external aerodynamic forces acting along the x-, y- and z-axes
- \( L, M, N \) are external aerodynamic moments about the x-, y- and z-axes
- \( u, v, w \) are translational velocity components of helicopter along fuselage x-, y- and z-axes
- \( p, q, r \) are angular velocity components of helicopter about fuselage x-, y- and z-axes
- \( \phi, \theta, \psi \) are Euler angles defining the orientation of the aircraft relative to the Earth
- \( \theta_0, \theta_{otr} \) are collective pitch angle
- \( \theta_{ls}, \theta_{lc}, \theta_{lw} \) are longitudinal and lateral cyclic pitch (subscript w denotes hub/wind axes)
- \( \phi_X, \phi_Y, \phi_Z \) are in-plane force coefficients
- \( C_T \) is rotor thrust coefficient
- \( \psi_w \) is rotor sideslip angle
- \( \chi_s \) is rotor shaft forward tilt angle
- \( \beta_{lc}, \beta_{ls}, \beta_{lw} \) are rotor blade coning, longitudinal and lateral flapping angles (subscript w denotes hub/wind axes)
- \( \rho \) is air density
- \( \Omega \) is rotor speed
- \( R \) is rotor radius
- \( C_L, C_M, C_Q \) are moment coefficients
- \( K_\beta \) is center-spring rotor stiffness
- \( N_b \) is number of blades on main rotor
- \( \lambda_0, \lambda_{1c}, \lambda_{1w} \) are rotor uniform and first harmonic inflow velocities
- \( \mu \) is advance ratio
- \( \mu_x, \mu_y, \mu_z \) are velocities of the rotor hub in hub/shaft axes
\[ \alpha_0 \quad \text{main rotor blade lift curve slope (1/rad)} \\
\delta \quad \text{rotor solidity} \\
\theta_{tw} \quad \text{main rotor blade linear twist} \]

1. INTRODUCTION

There exists a growing demand for heavy vertical lift rotorcraft transportation systems. Most research and development focuses on the creation of larger, more powerful, and expensive lifting rotorcraft vehicles. These systems accomplish the task by scaling up existing technology to increase payload capability, but unfortunately at the same time increase complexity and cost [1]. An alternative approach would be use to smaller, existing helicopters working cooperatively to perform the same task. However, in multi-lift system configuration shown in fig. 1, cable forces applied on each helicopter have significant variation, which dramatically affect the characteristics of those helicopters’ flight dynamics, such as trim, stability and response. Thus, using smaller helicopters place more emphasis on the control of the individual helicopter besides the team strategy and control [2].

The twin-lift concept has been in existence in the helicopter industry for more than four decades [3]. The primary motivation is to improve the helicopter payload capacity while avoiding the need for building larger and larger vehicles. It has been shown that beyond a certain payload capacity, the economics of the helicopter performance obeys the law of diminishing returns [4].

References [5-7] focused on developing twin-lift or multi-lift system models. Manual control aspects of a twin-lift configuration have been studied in considerable detail. The pilot opinion of the flying qualities of the twin-lift configuration in a completely manual control mode has not been favorable, which is primarily due to the significant increase in cockpit workload. The reference [8] also pointed out that a conventional stability augmentation system was not adequate for precision hover and load release, and could result in pilot-induced oscillations (PIO). A more effective solution consisted of an inner loop in which the relative motion of aircraft and load was fed back to cyclic, and an outer loop in which the aircraft position above ground was fed back again to cyclic.

In references [9-10], the feed forward controller was designed to avoid excitation of the lightly damped modes of the system by shaping the reference trajectory using robust inputs. These studies had dealt with the dynamic properties of the linearized twin-lift system about steady-state hover conditions. In Reference [11], the controller was a combined feed-forward and feedback scheme for simultaneous avoidance of swing excitation and active swing damping. Laboratory flight tests show the effectiveness of the combined control system. In reference [12-16], a nonlinear control technique using state feedback with a reduced order model of the twin-lift system was presented. These work researched the closed loop performance of nonlinear controllers for the two twin-lift configurations and the spreader bar configuration. These references suggested to use a state-feedback based nonlinear automatic controller to provide stability.
augmentation to the system, which will also simplify the piloting of the twin-lift task. The control technique is based on feedback linearization [17] which has several advantages over controller designs based on linearized dynamics. Reference [18] adopted multivariable LQG/LTR design methodology to design the controller of twin-lift helicopter system. In order to improve handling qualities for helicopters with slung loads, reference [19] used cable angle feed back to design controller improving helicopter control response.

In summary, it is very difficult to directly introduce cable forces feedback in controller design due to lack of explicit physical transformation from forces to actuator control. Therefore, forgoing methods were all based on helicopters’ linear trim models to design and evaluate controllers without considering cable forces direct feedback. In this presented work, different dynamic properties of each helicopter in multi-lift system are considered. A new control method is presented based on aerodynamic inversion, in which forces and moments of the payload directly feed into nonlinear controller. The most significant improvement is that each helicopter in twin-lift helicopter has the same control structure and control law, which remarkably simplify design of controller and improve the control performance.

2 NONLINEAR MODEL OF TWIN LIFT HELICOPTER SYSTEM

A general simulation model of multi-lift helicopter system with elastic cable has been developed for research on control and avionic instrumentation. Rather than treating the aircraft and load as a multi-body system, they were treated as separate objects each with its own independent equations of motion, which operate separately, but synchronously. The objects in multi-lift system were coupled exclusively through the slings’ forces that act on all bodies. Based on this model, trim and stability results and analyses of twin lift system could be conducted. The following figures show the stability results of two type of twin lift helicopter system with different payload weight and different forward speed.

From the eigenvalue of two type of twin lift system, we can conclude that:

* Helicopter has different dynamic properties with speed variation and weight of payload;
* The stability will change significantly.
with flight speed and the weight of payload;
* Every helicopter has the different time response under the same input command, and it is a challenge to coordinate control; According to these results, controller should
* Guarantee each helicopter has identical stability and response; while having same control structure to simplify the controller design,
* Cancel forces and moments generated by payload that affects greatly on individual helicopter’s dynamic properties.
* Have potential capability to reduce the slung load swing during flight.

3 NONLINEAR CONTROLLER

3.1 Nonlinear dynamic inverse control

The primary control objective of the twin-lift system controllers is to enable two helicopters to carry the load to the desired destination in 3D space within a specified time-interval. It is vitally important that this maneuver should be carried out while maintaining safe separation between the helicopters to keep their rotor discs apart. This can be achieved by keeping the separations close to some reference values. Inverse dynamic algorithms for rotorcraft control generally adhere to one of two forms, the differentiation method and the integration method. A classical rotorcraft non-linear dynamic system can be defined from the following input-output,

\[
\dot{x} = f(x,u), x = x(0) \\
y = g(x)
\]

(1)

(2)

**Differentiation Method**

The differentiation method may be illustrated more clearly by differentiating equation (2):

\[
\dot{y} = \frac{\partial g(x)}{\partial x} \dot{x} = \frac{\partial g(x)}{\partial x} f(x,u) \\
\vdots \\
y' = \alpha(x) + \beta(x)u
\]

(3)

When the control terms \(u\) appear explicitly in \(y'\) derivative in equation (3), define \(v = y'\), where \(v\) is pseudo control. We have an \(r\)-integrator linear system: \(v = y'\). We can now design a controller for this system by using any linear control method. At the same time, we have

\[
v = \alpha(x) + \beta(x)u \tag{4}
\]

The controller that is implemented and obtained through

\[
u = \beta(x)^{-1}(v - \alpha(x)) \tag{5}
\]

Any linear method can be used to design \(v\). Forgoing equations present a simple description of the inverse control problem under certain conditions. The system is driven by the rate of change of the output vector and produces a time history of control displacements required to generate the desired output vector by integrating between two points in time.

**Integration Method**

The integration method of inverse simulation has been exhaustively investigated [20-24] for various applications in helicopter researches. Unlike differentiation algorithms, integration-based inverse simulation algorithms can operate independently on the helicopter model. Assuming all initial conditions have been correctly assigned, the output vector can be approximated by

\[
x(t_{k+1}) = \Phi(x(t_k),u(t_k)) \\
y(t_{k+1}) = g(x(t_{k+1})) \tag{6}
\]

The mapping from input \(u(t_k)\) to output \(y(t_{k+1})\) is approximated by the non-linear rotorcraft simulation model. As the desired output vector \(y_d(t_{k+1})\) is known, an error function can be constructed with equation below,

\[
e(t_{k+1}) = y_d(t_{k+1}) - y(t_{k+1}) = y_d(t_{k+1}) - g(x(t_{k+1})) \tag{7}
\]

Finding the available controls at time \(t_k\),
which will set the error function to zero, will ensure that the maneuver constraints are met. Continued application of this procedure during the entire duration of the maneuver will yield a time history of the required control inputs. The Newton-Raphson iteration is used to reduce the error to zero using equation,

\[ u(t_{k+1}) = u(t_k) - [J(t_k)]^{-1} e(t_k) \]

where \( J \) is the Jacobian matrix. The iteration loop is terminated, when \( e(t_{k+1}) < tol \), where \( tol \) is a preset tolerance, which reflect the demanded accuracy of the simulation. In general, the tolerance limits have a significant bearing on the results of the inverse simulation.

Generally, both dynamic inverse methods have shortcomings respectively in twin lift system control law design. For differentiation method, it is very hard to obtain a perfect model with perfect derivatives for complicated helicopter flight dynamics. In fact, the rotorcraft inverse model is generally based on trim linearization square model, and this will result in inverse dynamics control errors. For integral method, it is very difficult to apply this method for helicopter flight control because of iteration’s efficiency. The most remarkable shortcoming of these two methods in twin-lift system control is that neither of them can handle measured variable forces and moments generated from slung load. These forces and moments have great influence on each helicopter’s flight dynamic properties. In current work, a new method is presented. It uses the idea of these two methods as a reference, but has capabilities to directly handle measured forces and moments.

### 3.2 Nonlinear Controller Synthesis Based on Control Input Inverse Solution

The start point of the method in this paper is that helicopters are treated as a real physical system. It just likes a human pilot controlling the helicopter. When pilots want to control helicopter to meet desired states, they give four control commands. At the same time, they continue to provide correct control inputs according to current flight states. The following figure shows the idea.

From mathematical model and simulation stand, helicopter model includes two components, which are aerodynamic force module and equation of motion module. The pilots’ model can be treated as an intelligent controller, which includes inverse equation of motion module and inverse aerodynamic force module. If aerodynamic force module and inverse aerodynamic module are reciprocal, and equation of motion module and inverse equation of motion module are reciprocal too, then current states will follow desired states in forgoing system. How to establish these two inverse models are the key point and are explained as follow.

Helicopter flight mechanics model can be written as,

\[ X = M(\dot{\mathbf{u}} - r\mathbf{v} + q\mathbf{w}) \]
\[ Y = M(\dot{\mathbf{v}} - p\mathbf{w} + r\mathbf{u}) \]
\[ Z = M(\dot{\mathbf{w}} - q\mathbf{u} + p\mathbf{v}) \]
\[ L = I_{xx} \dot{\mathbf{p}} - (I_{yy} - I_{zz})qr - I_{xz}(pq + \dot{r}) \]
\[ M = I_{yy} \dot{\mathbf{q}} - (I_{zz} - I_{xx})pr + I_{yz}(p^2 - r^2) \]
\[ N = I_{zz} \dot{\mathbf{r}} - (I_{xx} - I_{yy})pq - I_{yx}(\mathbf{p} + qr) \]

Fig 6 procedure of control input inverse solution
\[ p = \dot{\phi} - \psi \sin \theta \]
\[ q = \dot{\theta} \cos \phi + \psi \sin \phi \cos \theta \]
\[ r = -\dot{\theta} \sin \phi + \psi \cos \phi \cos \theta \]

The right hand sides of equations (8-10) stand for the equation of motion in figure 6.

And, the left hand sides of equations (8-9) can be written,

\[ X = X_{\text{req}}(x, \psi, \phi) + X_{\text{frt}}(x, \psi, \phi) + X_{\text{trt}}(x, \psi, \phi) + X_{\text{hth}}(x, \psi, \phi) + X_{\text{ver}}(x, \psi, \phi) \]
\[ Y = Y_{\text{req}}(x, \psi, \phi) + Y_{\text{frt}}(x, \psi, \phi) + Y_{\text{trt}}(x, \psi, \phi) + Y_{\text{hth}}(x, \psi, \phi) + Y_{\text{ver}}(x, \psi, \phi) \]
\[ Z = Z_{\text{req}}(x, \psi, \phi) + Z_{\text{frt}}(x, \psi, \phi) + Z_{\text{trt}}(x, \psi, \phi) + Z_{\text{hth}}(x, \psi, \phi) + Z_{\text{ver}}(x, \psi, \phi) \]
\[ L = L_{\text{req}}(x, \psi, \phi) + L_{\text{frt}}(x, \psi, \phi) + L_{\text{trt}}(x, \psi, \phi) + L_{\text{hth}}(x, \psi, \phi) + L_{\text{ver}}(x, \psi, \phi) \]
\[ M = M_{\text{req}}(x, \psi, \phi) + M_{\text{frt}}(x, \psi, \phi) + M_{\text{trt}}(x, \psi, \phi) + M_{\text{hth}}(x, \psi, \phi) + M_{\text{ver}}(x, \psi, \phi) \]
\[ N = N_{\text{req}}(x, \psi, \phi) + N_{\text{frt}}(x, \psi, \phi) + N_{\text{trt}}(x, \psi, \phi) + N_{\text{hth}}(x, \psi, \phi) + N_{\text{ver}}(x, \psi, \phi) \]

The above equations are shown as the aerodynamic force module in figure 6.

Where \( x = (u, v, w, p, q, r, \phi, \theta, \psi, \text{rotor state})^T \) is helicopter’s state, \( u_1 = [\theta, \phi, \psi]^T \), \( u_2 = [\theta_w] \) are helicopter’s control inputs. The subscripts stand for: rotor, r; tail rotor, tr; fuselage, f; horizontal tail plane, h; and vertical fin, v; gravity, g; cable (sling), c.

Equations (8-11) are aerodynamic force and motion of equation module in figure 6. In equation (11), except main rotor and tail rotor, aerodynamic forces of all other components basically depend on the flight states. The flight mechanics modeling has been based on empirical fitting of wind tunnel test data, gathered at a limited range of dynamic pressure and fuselage angles of incidence at model or full scale. The forces at a general flight speed, or dynamic pressure, can be estimated from the data at the measured conditions through the relationship

\[ \frac{F_{\text{test}}(\rho, \gamma)}{\rho S} = \frac{F_{\text{ref}}(\rho, \gamma) \rho^2 S}{\rho_{\text{ref}} S \rho_{\text{ref}}} \]  

Where the subscript test refers to the tunnel test conditions and S is a reference area. So forces and moments generated from fuselage, horizontal stabilizer, vertical fin, and landing gear are all known based on current flight states feedback. In addition, forces and moments generated from slings of payload can be measured directly by instrumentation system. So, all forces and moments are known based on current states feedback except those generated from main rotor and tail rotor due to control inputs dependence.

If we have the desired commands \( [z_r, \phi_r, \theta_r, \psi_r]^T \) and their derivatives, according to equation (10), it is very easy to get \( [w_r, p_r, q_r, r_r]^T \) and their derivatives. Considering equations (11), we have,

\[ X_{\text{req}}(x, \psi, \phi) = M(\dot{u} - \dot{r}v + qw - w_r) \]
\[ -X_{\text{frt}}(x, \psi, \phi) - X_{\text{trt}}(x, \psi, \phi) - X_{\text{hth}}(x, \psi, \phi) - X_{\text{ver}}(x, \psi, \phi) \]
\[ Y_{\text{req}}(x, \psi, \phi) + Y_{\text{frt}}(x, \psi, \phi) + Y_{\text{trt}}(x, \psi, \phi) + Y_{\text{hth}}(x, \psi, \phi) + Y_{\text{ver}}(x, \psi, \phi) \]
\[ Z_{\text{req}}(x, \psi, \phi) + Z_{\text{frt}}(x, \psi, \phi) + Z_{\text{trt}}(x, \psi, \phi) + Z_{\text{hth}}(x, \psi, \phi) + Z_{\text{ver}}(x, \psi, \phi) \]
\[ L_{\text{req}}(x, \psi, \phi) + L_{\text{frt}}(x, \psi, \phi) + L_{\text{trt}}(x, \psi, \phi) + L_{\text{hth}}(x, \psi, \phi) + L_{\text{ver}}(x, \psi, \phi) \]
\[ M_{\text{req}}(x, \psi, \phi) + M_{\text{frt}}(x, \psi, \phi) + M_{\text{trt}}(x, \psi, \phi) + M_{\text{hth}}(x, \psi, \phi) + M_{\text{ver}}(x, \psi, \phi) \]
\[ N_{\text{req}}(x, \psi, \phi) + N_{\text{frt}}(x, \psi, \phi) + N_{\text{trt}}(x, \psi, \phi) + N_{\text{hth}}(x, \psi, \phi) + N_{\text{ver}}(x, \psi, \phi) \]

The right hand side of equations is known. These equations indicate that desired forces and moments generated from main rotor and tail rotor to realize desired flight states are known under current and desired states. In other word, in order to realize desired states, forces and moments generated by the main and tail rotor can be obtained depending on the current states and desired states. Substitute equations (12) into equations (8, 9), we can obtain the following equations

\[ M(\ddot{u} - \dot{r}v + qw - w_r) = M(\ddot{u} - \dot{r}v + qw) \]
\[ M(\ddot{v} - \dot{p}w + \dot{r}u) = M(\ddot{v} - \dot{p}w + \dot{r}u) \]
\[ M(u_1, q - u + p v) = M(u_1, q - u + p v) \]
\[ I_{\text{ref}}(p + r) = I_{\text{ref}}(p + r) - I_{\text{ref}}(p + r) + I_{\text{ref}}(p + r) \]
\[ I_{\text{ref}}(p + r) = I_{\text{ref}}(p + r) - I_{\text{ref}}(p + r) + I_{\text{ref}}(p + r) \]
\[ I_{\text{ref}}(p + r) = I_{\text{ref}}(p + r) - I_{\text{ref}}(p + r) + I_{\text{ref}}(p + r) \]

There are six independent equations in above formulas, and the following equations are solutions of (13).
Define $v=[\tilde{z}, \tilde{\phi}, \tilde{r}, \tilde{\psi}]^T$, $v$ is pseudo control vector. Therefore, the complicated dynamic system has been transformed into a linear system, and we can now design a controller for this system by using any linear controller design method. Advantages of this research idea are that: Control law design scheme is similar to feedback linearization method.

Measured external forces and moments applied on the helicopter can directly feed into nonlinear controllers to make each helicopter have same stability and response in multi-lift system.

### 3.3 Control Input Inverse Solution

Control Input Direct Solution modeling described in here pays more attention on the key aerodynamic features, which dramatically affects the helicopter's flight characteristics.

#### 3.3.1 Forces and moments of main rotor.

Main rotor forces in body axis can be written as:

$$
\begin{bmatrix}
X_r \\
Y_r \\
Z_r
\end{bmatrix} = Th2b \cdot Tw2h 
\begin{bmatrix}
CX_{hw}(\rho \Omega^2 R^4) \\
CY_{hw}(\rho \Omega^2 R^4) \\
-CT(\rho \Omega^2 R^4)
\end{bmatrix}
$$

Where matrix,

$$Th2b = 
\begin{bmatrix}
\cos \gamma_s & 0 & -\sin \gamma_s \\
0 & 1 & 0 \\
\sin \gamma_s & 0 & \cos \gamma_s
\end{bmatrix}
$$

is transformation matrix from hub axis to body axis, and matrix $Th2b = Th2b^T$ is transformation matrix from body axis to hub axis, $\gamma_s$, is rotor shaft forward tilt angle. Matrix,

$$Th2w = 
\begin{bmatrix}
\cos \gamma_w & -\sin \gamma_w & 0 \\
\sin \gamma_w & \cos \gamma_w & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

transformation matrix from hub/wind system to hub system, and matrix $Th2w = Th2w^T$, is transformation matrix from hub system to hub/wind system, $\gamma_w$, is rotor sideslip angle.

The rotor moments can be expressed as

$$CX_{hw}$$ and $$CY_{hw}$$ are in-plane force coefficients which result from a combination of multitude of physical effects. In general, in-plane forces include two main components, first components are the first harmonics of the product of the lift and flapping in the direction of motion and represent the contribution to X and Y from blades in the fore and aft positions; second components represent the contributions to X and Y from the induced and profile drag acting on the advancing and retreating blades. In helicopter moderate speed, the combination of these effects reduces to the simple result, and the hub forces are given entirely by the tilt of the rotor thrust vector due to the cancellation of in-plane contributions from the blade lift forces [25],

$$
\begin{bmatrix}
CX_{hw} \\
CY_{hw}
\end{bmatrix} = 
\begin{bmatrix}
CT \cdot \beta_{iw} \\
-CT \cdot \beta_{iw}
\end{bmatrix}
$$

The rotor forces then can be expressed as

$$
\begin{bmatrix}
X_r \\
Y_r \\
Z_r
\end{bmatrix} = Th2b \cdot Tw2h 
\begin{bmatrix}
CT \cdot \beta_{iw}(\rho \Omega^2 R^4) \\
-CT \cdot \beta_{iw}(\rho \Omega^2 R^4) \\
-CT(\rho \Omega^2 R^4)
\end{bmatrix}
$$

The rotor moments can be expressed as
According to main rotor forces and moments, we can directly give tail rotor forces and moments in body axis:

\[
\begin{align*}
X_{tr} &= 0 \\
Y_{tr} &= CT_{tr} \rho \Omega^2 R_{tr}^4 \\
Z_{tr} &= 0 \\
L_{tr} &= h_{tr} CT_{tr} \rho \Omega^2 R_{tr}^4 \\
M_{tr} &= C_{Qtr} \rho \Omega^2 R_{tr}^4 \\
N_{tr} &= -I_{tr} CT_{tr} \rho \Omega^2 R_{tr}^4
\end{align*}
\]  

(26)

3.3.3 Forces and moments of main rotor and tail rotor

Forces and moments of main rotor and tail rotor can be written as:

\[
\begin{align*}
X_{m} &= CT \cdot \beta_{m} \cdot (\rho \Omega^2 R_{m}^4) \\
Y_{m} &= CT \cdot \beta_{m} \cdot (\rho \Omega^2 R_{m}^4) + CT_{m} \rho \Omega^2 R_{m}^4 \\
Z_{m} &= CT \cdot \beta_{m} \cdot (\rho \Omega^2 R_{m}^4) \\
L_{m} &= C_{Lm} \rho \Omega^2 R_{m}^4 \\
M_{m} &= C_{Qm} \rho \Omega^2 R_{m}^4 \\
N_{m} &= -I_{m} CT \cdot \beta_{m} \cdot (\rho \Omega^2 R_{m}^4)
\end{align*}
\]  

(27)

In order to meet desired commands \([z, \phi, \theta, w]^T\), we substitute desired commands and current flight state into equation (12), and calculate aerodynamics of other components, then desired forces and moments of main rotor and tail rotor can be obtained, \([X_{m}, Y_{m}, Z_{m}, L_{m}, M_{m}, N_{m}]^T\). Of course, desired forces \([X_{tr}, Y_{tr}, Z_{tr}]^T\) can be obtained too. And they can act as initial value in the following control input solution.

From equations (27, 28), control inputs, \([\beta_{m}, \beta_{tr}, \beta_{m}, \beta_{tr}]^T\), did not explicitly manifest. In order to solve control inputs, parameters \(CT, C_{Qm}, CT_{m}, C_{Qtr}, \beta_{m}, \beta_{tr}\) need to be obtained first. At same time, other parameters, such as \(h_{tr}, I_{tr}, \rho, \Omega, \beta_{m}, \beta_{tr}\) also need to be solved. The number of unknown variables is greater than that of equations. The
following description shows how to get desired control input vector \([\alpha, \theta, \alpha, \phi, \psi, \alpha, \omega, \phi, \psi, \omega]\) using equations (27, 28). Firstly, from equation (27), \(Z_{\omega}\) is known, it has only direct relationship with \(C_T\). Therefore, thrust coefficient, \(C_T\), can be solved. As soon as these variables are known, they can act as the initial value in equations (28). Then we comply the following procedure, we can obtain control inputs step by step. \(Z_{\omega} \Rightarrow Z_\omega \Rightarrow \lambda_0 \Rightarrow C_Q \Rightarrow T_\nu \Rightarrow \lambda_{\omega} \Rightarrow C_{Q\omega} \Rightarrow \beta_{\omega r}, \beta_{\omega l} \Rightarrow \theta_{\omega r}, \theta_{\omega l} = \theta_{\omega r}, \theta_{\omega l}, \theta_{\omega r}\).

3.3.4 Control Input solution
According to Momentum theory in forward flight, the rotor inflow and thrust have tight relationship. They affect with each other. The general expression is given by normalizing velocities and rotor thrust in the usual way,

\[
\lambda_0 = \frac{C_T}{2\sqrt{\mu^2 + (\lambda_0 - \mu)^2}}
\]  

(29)

Where \(\mu = V \cos \alpha_s / \Omega R\), \(\mu = V \sin \alpha_s / \Omega R\). From equation (27), thrust coefficient \(C_T\) can be solved first. Because the inflow depends on the thrust and the thrust depends on the inflow, a solution is required to obtain rotor inflow \(\lambda_0\) based on above equation (29).

3.3.5 Rotor torque moment
The torque moment, approximated by the yawing moment in the hub/wind axes, can be obtained by integrating the moments of the in-plane loads about the shaft axis. According to ref. 25, a considerably simpler, but very effective, approximation to neglect small terms leads to the final equation for rotor aerodynamic torque can be derived. Rotor torque coefficient can then be written as

\[
\frac{2C_Q}{\alpha_s} = -(\mu_\omega - \lambda_0)(\frac{2C_T}{\alpha_s}) + \mu(\frac{2C_X_{\omega}}{\alpha_s}) + \frac{\delta}{4\alpha_0}(1 + 3\mu^2)
\]  

(30)

Then \(C_Q\) can be solved using above equation. Now \(C_T, C_Q\), and \(\lambda_0\) are set. According to the known helicopter torque equation (28) \(\lambda_{\omega r}\), tail rotor trust coefficient \(C_{T\nu}\) can be obtained, then tail rotor inflow \(\lambda_{\omega r}\), may be solved based on \(C_{T\nu}\) like the solution of main rotor inflow.

Tail rotor trust coefficient can be written as:

\[
C_{T\nu} = \frac{\alpha_{\omega r}}{2}\left[\theta_{\omega r} \frac{1}{3} \frac{\mu^2}{2} + \frac{\mu_{\omega r} - \lambda_{\omega r}}{2} + \frac{4}{4}(1 + \mu_{\omega r})\right]
\]  

(31)

Based on this equation, control input \(\theta_{\omega r}\) can be solved. \(C_Q\) is obtained based on \(C_{T\nu}\) with same method used in main rotor. Now let's review other two equations of \(L_{\nu, M_{\nu, \omega}}\) in equation (28). The only unknown parameters are flapping angle, \(\beta_{\omega r}\), \(\beta_{\omega l}\). They can be solved using these two \(L_{\nu, M_{\nu, \omega}}\) equations.

Using formula \([\beta_{\omega r}, \beta_{\omega l}] = \frac{1}{2\Omega R}[\beta_{\omega r}]\), flapping angles \(\beta_{\omega r}\), \(\beta_{\omega l}\) can be obtained in hub coordinate.

At last, main rotor control inputs need to be solved. The quasi-steady flap equations and thrust coefficient equation can be used to calculate main rotor collective and cyclic control in the hub wind. The quasi-steady flap equations and thrust coefficient equation can be expressed as:

\[
\begin{bmatrix}
\beta_{\omega r} \\
\beta_{\omega l}
\end{bmatrix} = A_{\beta_0} \begin{bmatrix}
\theta_0 \\
\theta_{\omega r} \\
\theta_{1r} \\
\theta_{1w}
\end{bmatrix} + A_{\beta_\theta} \begin{bmatrix}
\mu_{\omega r} \\
\mu_{\omega l}
\end{bmatrix} \frac{1}{2\Omega R}[\beta_{\omega r}] + A_{\beta_{\omega r}} \begin{bmatrix}
\bar{\theta}_{1r} \\
\bar{\theta}_{1w}
\end{bmatrix}
\]

(32)

\[
\mu_{\omega r} = \frac{\alpha_{\omega r}}{2} \left[\frac{\theta_{\omega r}}{3 \frac{1}{3}} + \frac{\mu^2}{2} + \frac{\mu_{\omega r} - \lambda_{\omega r}}{2} + \frac{4}{4}(1 + \mu_{\omega r})\right]
\]  

(33)

The detail information of foregoing equations can be reviewed in reference [25]. At this time, there are 3 independent equations and 3 unknown variables, \(\theta_0, \theta_{1r}, \theta_{1w}\). As soon as these variables are...
calculated, using the equation
\[
\begin{bmatrix}
\theta_{1w} \\
\theta_{2w}
\end{bmatrix}
= Tw2h \begin{bmatrix}
\theta_{1w} \\
\theta_{2w}
\end{bmatrix}
\]
can solve main rotor longitudinal and lateral cyclic pitch.

4 OUTER LOOP CONTROLLER DESIGN

Equation (14) is a second order integral linear system. PID (proportional, integral, and derivative) control laws are formulated for each of the controlled variables in the vector by imposing,
\[
\begin{bmatrix}
\dot{z} + K_{p1}(z - \bar{z}) + K_{i1}(\bar{z} - \bar{z}) + K_{d1} \int (\bar{z} - \bar{z}) dt \\
\dot{\phi} + K_{p2}(\phi - \bar{\phi}) + K_{i2}(\bar{\phi} - \bar{\phi}) + K_{d2} \int (\bar{\phi} - \bar{\phi}) dt \\
\dot{\theta} + K_{p3}(\theta - \bar{\theta}) + K_{i3}(\bar{\theta} - \bar{\theta}) + K_{d3} \int (\bar{\theta} - \bar{\theta}) dt \\
\dot{\psi} + K_{p4}(\psi - \bar{\psi}) + K_{i4}(\bar{\psi} - \bar{\psi}) + K_{d4} \int (\bar{\psi} - \bar{\psi}) dt
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
K_D, K_P, K_I, \quad K_{d1}, K_{i1}, K_{d2}, K_{i2}, K_{d3}, K_{i3}, K_{d4}, K_{i4},
\]
are individual loop gains and can be designed to meet performance specifications in attitude control. Outer loop speed and position control laws can be designed with the same method based on attitude control loop.

The primary control objective of the twin lift controller is to enable the two helicopters to move the load to any desired destination within a specified time interval. This can be achieved by controlling the average positions of the helicopters. Each helicopter control structure shows in following figure 7.

Fig 7 Helicopter Control Structure

5 NUMERICAL SIMULATION

5.1 Flight Simulation

A twin lift simulation model in side by side configuration was developed. This model includes nine dynamic sub systems with two nonlinear helicopter flight dynamic models, two elastic cable dynamic subsystems, two nonlinear aerodynamic inverse subsystems, two linear PID dynamic subsystems, and one payload nonlinear dynamic subsystem. When the system control modes are yaw hold, altitude hold, lateral position hold and forward speed tracking, we give a set of approximate initial conditions which are shown in Table 1. Forward speed signal is 1\*t(ft/s) at the time of 50 second, and speed holds at the time of 100 second.

Table 1 Simulation model initial conditions

<table>
<thead>
<tr>
<th>Initial condition</th>
<th>Heli 1</th>
<th>Heli 2</th>
<th>Payload</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial condition</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>uwv in body axis (ft/s)</td>
<td>[0;0;0]</td>
<td>[0;0;0]</td>
<td>[0;0;0]</td>
</tr>
<tr>
<td>Angle rate (rad/s)</td>
<td>[0;0;0]</td>
<td>[0;0;0]</td>
<td>[0;0;0]</td>
</tr>
<tr>
<td>Eular Angle (rad)</td>
<td>[-0.0382; 0.0013;0]</td>
<td>[-0.0382; 0.0013;0]</td>
<td>[0;0;0]</td>
</tr>
<tr>
<td>uwv in NED (ft/s)</td>
<td>[0;0;0]</td>
<td>[0;0;0]</td>
<td>[0;0;0]</td>
</tr>
<tr>
<td>Position in NED (ft)</td>
<td>[0;0;0]</td>
<td>[0;40;0]</td>
<td>[0;20;53]</td>
</tr>
<tr>
<td>Hook point in body axis (ft)</td>
<td>[0;0;4]</td>
<td>[0;0;4]</td>
<td>[0;1;1] &amp; [0;1;-1]</td>
</tr>
<tr>
<td>Control input (deg)</td>
<td>[0.7202; 0.0741; 16.1925; 0.1960]</td>
<td>[0.7202; 0.0741; 16.1925; 10.1960]</td>
<td>[0;1;1]</td>
</tr>
<tr>
<td>Weight (lb)</td>
<td>3200</td>
<td>3200</td>
<td>3200</td>
</tr>
</tbody>
</table>

Payload was suspend by two elastic cables, whose stiffness constants and damping coefficients are 1000 pound/ft and 300 pound*ft/s.

The initial Euler angles and control inputs are generated from each helicopter's trim value without payload. The initial position of payload is approximate value. The simulation results shows as following figure 8.

8.1 Speed in fixed body frame of helicopter 1 time response 
8.2 Speed in fixed body frame of helicopter 2 time response
From the simulation result, in prior 50 seconds, control laws automatically find a new trim point for twin-lift system under approximate initial conditions. This result indicates that control laws have good disturbance rejection capabilities due to cable forces direct feedback, even initial conditions are not precise. It is reasonable that attitude angles and control inputs of two helicopters significantly change to support the payload.

During 50 -100 seconds, the control signals, which are 1ft/s² forward acceleration, lateral position, altitude and yaw angle remain as initial values, are given. Due to direct measured payload forces feedback, the nonlinear controllers are capable to predict the control input in collective pitch and longitudinal cyclic, and the forward speed response has good performance.

After 100 seconds, control mode is forward speed hold. From the simulation results, we find that damping coefficients
of the payload in roll axis are insufficient, and roll oscillation of the payload converge slowly which lead to helicopter heave and yaw oscillation. The results have good physical meaning. From the simulation results, the controllers' performance is acceptable. Controllers based on direct forces feedback guarantee each helicopter's identical stability and response, at same time, have the same control structure and control gains to simplify the controller design in twin-lift system.

5.2 Robust Performance Simulation

In order to evaluate the robust performance of the nonlinear controller, the weight of payload is increased to 6400lb, which is two times of original weight. Sensor noise of p q r angular rate which is only put on helicopter 1, is simulated by limited width band white noise (shown in fig 9). Control commands, control gains, and control structure are same as above simulation. Robust performance simulation results are shown in fig 10.

![Fig 9 Helicopter 1 sensor noise of p q r](image)

![10.1 Speed in fixed body frame of helicopter 1 time response](image)

![10.2 Speed in fixed body frame of helicopter 2 time response](image)

![10.3 Angular rate of helicopter 1 time response](image)

![10.4 Angular rate of helicopter 2 time response](image)

![10.5 Euler angle of helicopter 1 time response](image)

![10.6 Euler angle of helicopter 2 time response](image)

![10.7 Speed in NED frame of helicopter 1 time response](image)

![10.8 Speed in NED frame of helicopter 2 time response](image)

![10.9 Position in NED frame of helicopter 1 time response](image)

![10.10 Position in NED frame of helicopter 2 time response](image)

![10.11 Control input time history of helicopter 1](image)

![10.12 Control input time history of helicopter 2](image)

![10.13 Speed in fixed body frame of payload time response](image)

![10.14 Helicopters time response difference about speed](image)

![10.15 Angular rate of payload time response](image)

![10.16 Helicopters time response difference about angular rate](image)
The robust performance simulation results are acceptable under effects of payload weight change and sensor noise. Each helicopter can track control commands well. It is reasonable that collective pitch, longitudinal and lateral cyclic, and tail rotor pitch obviously change with weight increase of payload comparing with original simulation.  

6. CONCLUSION

A new control method which uses control input direct solution based on quasi-steady aerodynamics is presented in this paper. The method:

* Can transfer a complicated nonlinear rotorcraft control dynamic system into a linear system.

* Can directly handle the measured external forces applied on the helicopter, and make helicopters have same stability and response by using forces feedback.

* Can guarantee each helicopter's identical control structure and control gains to simplify control laws design of twin-lift system.

REFERENCES


